10.2.2 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 10 materials.

In Exercises 1 - 20, find the exact value of the cosine and sine of the given angle.

For help with these exercises, click one or more of the resources below:

- Defining cosine and sine using the Unit Circle
- 'Tricks' to help remember points on the Unit Circle
- Understanding 'reference' angles

1. $\theta = 0$
2. $\theta = \dfrac{\pi}{4}$
3. $\theta = \dfrac{\pi}{3}$
4. $\theta = \dfrac{\pi}{2}$
5. $\theta = \dfrac{2\pi}{3}$
6. $\theta = \dfrac{3\pi}{4}$
7. $\theta = \pi$
8. $\theta = \dfrac{7\pi}{6}$
9. $\theta = \dfrac{5\pi}{4}$
10. $\theta = \dfrac{4\pi}{3}$
11. $\theta = \dfrac{3\pi}{2}$
12. $\theta = \dfrac{5\pi}{3}$
13. $\theta = \dfrac{7\pi}{4}$
14. $\theta = \dfrac{23\pi}{6}$
15. $\theta = -\dfrac{13\pi}{2}$
16. $\theta = -\dfrac{43\pi}{6}$
17. $\theta = -\dfrac{3\pi}{4}$
18. $\theta = -\dfrac{\pi}{6}$
19. $\theta = \dfrac{10\pi}{3}$
20. $\theta = 117\pi$

In Exercises 21 - 30, use the results developed throughout the section to find the requested value.

21. If $\sin(\theta) = -\dfrac{7}{25}$ with θ in Quadrant IV, what is $\cos(\theta)$?
22. If $\cos(\theta) = \dfrac{4}{9}$ with θ in Quadrant I, what is $\sin(\theta)$?
23. If $\sin(\theta) = \dfrac{5}{13}$ with θ in Quadrant II, what is $\cos(\theta)$?
24. If $\cos(\theta) = -\dfrac{2}{11}$ with θ in Quadrant III, what is $\sin(\theta)$?
25. If $\sin(\theta) = -\dfrac{2}{3}$ with θ in Quadrant III, what is $\cos(\theta)$?
26. If $\cos(\theta) = \dfrac{28}{53}$ with θ in Quadrant IV, what is $\sin(\theta)$?
27. If $\sin(\theta) = \dfrac{2\sqrt{5}}{5}$ and $\dfrac{\pi}{2} < \theta < \pi$, what is $\cos(\theta)$?
28. If $\cos(\theta) = \dfrac{\sqrt{10}}{10}$ and $2\pi < \theta < \dfrac{5\pi}{2}$, what is $\sin(\theta)$?

29. If $\sin(\theta) = -0.42$ and $\pi < \theta < \dfrac{3\pi}{2}$, what is $\cos(\theta)$?

30. If $\cos(\theta) = -0.98$ and $\dfrac{\pi}{2} < \theta < \pi$, what is $\sin(\theta)$?

In Exercises 31 - 39, find all of the angles which satisfy the given equation.

31. $\sin(\theta) = \dfrac{1}{2}$

32. $\cos(\theta) = -\dfrac{\sqrt{3}}{2}$

33. $\sin(\theta) = 0$

34. $\cos(\theta) = \dfrac{\sqrt{2}}{2}$

35. $\sin(\theta) = \dfrac{\sqrt{3}}{2}$

36. $\cos(\theta) = -1$

37. $\sin(\theta) = -1$

38. $\cos(\theta) = \dfrac{\sqrt{3}}{2}$

39. $\cos(\theta) = -1.001$

In Exercises 40 - 48, solve the equation for t. (See the comments following Theorem 10.5.)

40. $\cos(t) = 0$

41. $\sin(t) = -\dfrac{\sqrt{2}}{2}$

42. $\cos(t) = 3$

43. $\sin(t) = -\dfrac{1}{2}$

44. $\cos(t) = \dfrac{1}{2}$

45. $\sin(t) = -2$

46. $\cos(t) = 1$

47. $\sin(t) = 1$

48. $\cos(t) = -\dfrac{\sqrt{2}}{2}$

In Exercises 49 - 54, use your calculator to approximate the given value to three decimal places. Make sure your calculator is in the proper angle measurement mode!

49. $\sin(78.95^\circ)$

50. $\cos(-2.01)$

51. $\sin(392.994)$

52. $\cos(207^\circ)$

53. $\sin\left(\pi^\circ\right)$

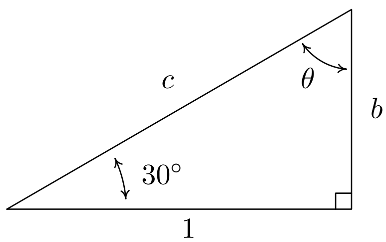
54. $\cos(e)$

In Exercises 55 - 58, find the measurement of the missing angle and the lengths of the missing sides. (See Example 10.2.8)

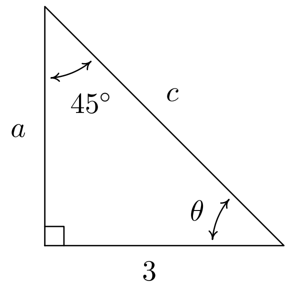
For help with these exercises, click the resource below:

- Introduction to right triangle trigonometry

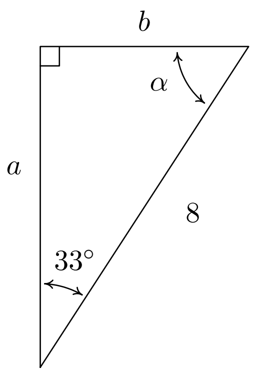
55. Find θ, b, and c.



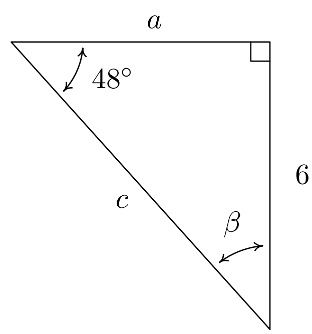
56. Find θ, a, and c.



57. Find α, a, and b.



58. Find β, a, and c.



In Exercises 59 - 64, assume that θ is an acute angle in a right triangle and use Theorem 10.4 to find the requested side.

For help with these exercises, click the resource below:

- Introduction to right triangle trigonometry

59. If $\theta = 12°$ and the side adjacent to θ has length 4, how long is the hypotenuse?

60. If $\theta = 78.123°$ and the hypotenuse has length 5280, how long is the side adjacent to θ?

61. If $\theta = 59°$ and the side opposite θ has length 117.42, how long is the hypotenuse?

62. If $\theta = 5°$ and the hypotenuse has length 10, how long is the side opposite θ?

63. If $\theta = 5°$ and the hypotenuse has length 10, how long is the side adjacent to θ?

64. If $\theta = 37.5°$ and the side opposite θ has length 306, how long is the side adjacent to θ?

In Exercises 65 - 68, let θ be the angle in standard position whose terminal side contains the given point then compute $\cos(\theta)$ and $\sin(\theta)$.

65. $P(-7, 24)$
66. $Q(3, 4)$
67. $R(5, -9)$
68. $T(-2, -11)$

In Exercises 69 - 72, find the equations of motion for the given scenario. Assume that the center of the motion is the origin, the motion is counter-clockwise and that $t = 0$ corresponds to a position along the positive x-axis. (See Equation 10.3 and Example 10.1.5.)

69. A point on the edge of the spinning yo-yo in Exercise 50 from Section 10.1.

    Recall: The diameter of the yo-yo is 2.25 inches and it spins at 4500 revolutions per minute.

70. The yo-yo in exercise 52 from Section 10.1.

    Recall: The radius of the circle is 28 inches and it completes one revolution in 3 seconds.

71. A point on the edge of the hard drive in Exercise 53 from Section 10.1.

    Recall: The diameter of the hard disk is 2.5 inches and it spins at 7200 revolutions per minute.

72. A passenger on the Big Wheel in Exercise 55 from Section 10.1.

    Recall: The diameter is 128 feet and completes 2 revolutions in 2 minutes, 7 seconds.

73. Consider the numbers: 0, 1, 2, 3, 4. Take the square root of each of these numbers, then divide each by 2. The resulting numbers should look hauntingly familiar. (See the values in the table on 757.)

74. Let α and β be the two acute angles of a right triangle. (Thus α and β are complementary angles.) Show that $\sin(\alpha) = \cos(\beta)$ and $\sin(\beta) = \cos(\alpha)$. The fact that co-functions of complementary angles are equal in this case is not an accident and a more general result will be given in Section 10.4.

75. In the scenario of Equation 10.3, we assumed that at $t = 0$, the object was at the point $(r, 0)$. If this is not the case, we can adjust the equations of motion by introducing a 'time delay.' If $t_0 > 0$ is the first time the object passes through the point $(r, 0)$, show, with the help of your classmates, the equations of motion are $x = r\cos(\omega(t - t_0))$ and $y = r\sin(\omega(t - t_0))$.

Checkpoint Quiz 10.2

1. Find the cosine and sine of the following angles:

   (a) $\theta = -210^\circ$ (b) $\theta = \frac{5\pi}{2}$ (c) $\theta = 405^\circ$ (d) $\theta = -\frac{4\pi}{3}$

2. Suppose α is a Quadrant II angle with $\sin(\alpha) = \frac{12}{13}$. Find $\cos(\alpha)$.
3. Suppose the terminal side of θ contains the point $(3, -4)$. Find $\cos(\theta)$ and $\sin(\theta)$.
4. Find all angles θ which satisfy $\sin(\theta) = -\frac{1}{2}$.
5. Find all real numbers t which satisfy $\cos(t) = 0$.
6. The angle of inclination from the base of one of the pyramids of Giza to its apex is 52°. If the slant height of the pyramid (that is the height of the triangular faces) is 578 feet, how tall is it?

For worked out solutions to this quiz, click the links below:

- Quiz Solution Part 1
- Quiz Solution Part 2
- Quiz Solution Part 3

10.2.3 Answers

1. $\cos(0) = 1,\ \sin(0) = 0$
2. $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2},\ \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
3. $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2},\ \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$
4. $\cos\left(\frac{\pi}{2}\right) = 0,\ \sin\left(\frac{\pi}{2}\right) = 1$
5. $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2},\ \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$
6. $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2},\ \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$
7. $\cos(\pi) = -1,\ \sin(\pi) = 0$
8. $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2},\ \sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$
9. $\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2},\ \sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
10. $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2},\ \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$
11. $\cos\left(\frac{3\pi}{2}\right) = 0,\ \sin\left(\frac{3\pi}{2}\right) = -1$
12. $\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2},\ \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$
13. $\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2},\ \sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
14. $\cos\left(\frac{23\pi}{6}\right) = \frac{\sqrt{3}}{2},\ \sin\left(\frac{23\pi}{6}\right) = -\frac{1}{2}$
15. $\cos\left(-\frac{13\pi}{2}\right) = 0,\ \sin\left(-\frac{13\pi}{2}\right) = -1$
16. $\cos\left(-\frac{43\pi}{6}\right) = -\frac{\sqrt{3}}{2},\ \sin\left(-\frac{43\pi}{6}\right) = \frac{1}{2}$
17. $\cos\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2},\ \sin\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
18. $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2},\ \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$
19. $\cos\left(\frac{10\pi}{3}\right) = -\frac{1}{2},\ \sin\left(\frac{10\pi}{3}\right) = -\frac{\sqrt{3}}{2}$
20. $\cos(117\pi) = -1,\ \sin(117\pi) = 0$
21. If $\sin(\theta) = -\frac{7}{25}$ with θ in Quadrant IV, then $\cos(\theta) = \frac{24}{25}$.
22. If $\cos(\theta) = \frac{4}{9}$ with θ in Quadrant I, then $\sin(\theta) = \frac{\sqrt{65}}{9}$.
23. If $\sin(\theta) = \frac{5}{13}$ with θ in Quadrant II, then $\cos(\theta) = -\frac{12}{13}$.
24. If $\cos(\theta) = -\frac{2}{11}$ with θ in Quadrant III, then $\sin(\theta) = -\frac{\sqrt{117}}{11}$.
25. If $\sin(\theta) = -\frac{2}{3}$ with θ in Quadrant III, then $\cos(\theta) = -\frac{\sqrt{5}}{3}$.
26. If $\cos(\theta) = \frac{28}{53}$ with θ in Quadrant IV, then $\sin(\theta) = -\frac{45}{53}$.

27. If $\sin(\theta) = \dfrac{2\sqrt{5}}{5}$ and $\dfrac{\pi}{2} < \theta < \pi$, then $\cos(\theta) = -\dfrac{\sqrt{5}}{5}$.

28. If $\cos(\theta) = \dfrac{\sqrt{10}}{10}$ and $2\pi < \theta < \dfrac{5\pi}{2}$, then $\sin(\theta) = \dfrac{3\sqrt{10}}{10}$.

29. If $\sin(\theta) = -0.42$ and $\pi < \theta < \dfrac{3\pi}{2}$, then $\cos(\theta) = -\sqrt{0.8236} \approx -0.9075$.

30. If $\cos(\theta) = -0.98$ and $\dfrac{\pi}{2} < \theta < \pi$, then $\sin(\theta) = \sqrt{0.0396} \approx 0.1990$.

31. $\sin(\theta) = \dfrac{1}{2}$ when $\theta = \dfrac{\pi}{6} + 2\pi k$ or $\theta = \dfrac{5\pi}{6} + 2\pi k$ for any integer k.

32. $\cos(\theta) = -\dfrac{\sqrt{3}}{2}$ when $\theta = \dfrac{5\pi}{6} + 2\pi k$ or $\theta = \dfrac{7\pi}{6} + 2\pi k$ for any integer k.

33. $\sin(\theta) = 0$ when $\theta = \pi k$ for any integer k.

34. $\cos(\theta) = \dfrac{\sqrt{2}}{2}$ when $\theta = \dfrac{\pi}{4} + 2\pi k$ or $\theta = \dfrac{7\pi}{4} + 2\pi k$ for any integer k.

35. $\sin(\theta) = \dfrac{\sqrt{3}}{2}$ when $\theta = \dfrac{\pi}{3} + 2\pi k$ or $\theta = \dfrac{2\pi}{3} + 2\pi k$ for any integer k.

36. $\cos(\theta) = -1$ when $\theta = (2k+1)\pi$ for any integer k.

37. $\sin(\theta) = -1$ when $\theta = \dfrac{3\pi}{2} + 2\pi k$ for any integer k.

38. $\cos(\theta) = \dfrac{\sqrt{3}}{2}$ when $\theta = \dfrac{\pi}{6} + 2\pi k$ or $\theta = \dfrac{11\pi}{6} + 2\pi k$ for any integer k.

39. $\cos(\theta) = -1.001$ never happens

40. $\cos(t) = 0$ when $t = \dfrac{\pi}{2} + \pi k$ for any integer k.

41. $\sin(t) = -\dfrac{\sqrt{2}}{2}$ when $t = \dfrac{5\pi}{4} + 2\pi k$ or $t = \dfrac{7\pi}{4} + 2\pi k$ for any integer k.

42. $\cos(t) = 3$ never happens.

43. $\sin(t) = -\dfrac{1}{2}$ when $t = \dfrac{7\pi}{6} + 2\pi k$ or $t = \dfrac{11\pi}{6} + 2\pi k$ for any integer k.

44. $\cos(t) = \dfrac{1}{2}$ when $t = \dfrac{\pi}{3} + 2\pi k$ or $t = \dfrac{5\pi}{3} + 2\pi k$ for any integer k.

45. $\sin(t) = -2$ never happens

46. $\cos(t) = 1$ when $t = 2\pi k$ for any integer k.

47. $\sin(t) = 1$ when $t = \dfrac{\pi}{2} + 2\pi k$ for any integer k.

48. $\cos(t) = -\dfrac{\sqrt{2}}{2}$ when $t = \dfrac{3\pi}{4} + 2\pi k$ or $t = \dfrac{5\pi}{4} + 2\pi k$ for any integer k.

49. $\sin(78.95^\circ) \approx 0.981$

50. $\cos(-2.01) \approx -0.425$

51. $\sin(392.994) \approx -0.291$

52. $\cos(207^\circ) \approx -0.891$

53. $\sin(\pi^\circ) \approx 0.055$

54. $\cos(e) \approx -0.912$

55. $\theta = 60^\circ$, $b = \dfrac{\sqrt{3}}{3}$, $c = \dfrac{2\sqrt{3}}{3}$

56. $\theta = 45^\circ$, $a = 3$, $c = 3\sqrt{2}$

57. $\alpha = 57^\circ$, $a = 8\cos(33^\circ) \approx 6.709$, $b = 8\sin(33^\circ) \approx 4.357$

58. $\beta = 42^\circ$, $c = \dfrac{6}{\sin(48^\circ)} \approx 8.074$, $a = \sqrt{c^2 - 6^2} \approx 5.402$

59. The hypotenuse has length $\dfrac{4}{\cos(12^\circ)} \approx 4.089$.

60. The side adjacent to θ has length $5280\cos(78.123^\circ) \approx 1086.68$.

61. The hypotenuse has length $\dfrac{117.42}{\sin(59^\circ)} \approx 136.99$.

62. The side opposite θ has length $10\sin(5^\circ) \approx 0.872$.

63. The side adjacent to θ has length $10\cos(5^\circ) \approx 9.962$.

64. The hypotenuse has length $c = \dfrac{306}{\sin(37.5^\circ)} \approx 502.660$, so the side adjacent to θ has length $\sqrt{c^2 - 306^2} \approx 398.797$.

65. $\cos(\theta) = -\dfrac{7}{25}$, $\sin(\theta) = \dfrac{24}{25}$

66. $\cos(\theta) = \dfrac{3}{5}$, $\sin(\theta) = \dfrac{4}{5}$

67. $\cos(\theta) = \dfrac{5\sqrt{106}}{106}$, $\sin(\theta) = -\dfrac{9\sqrt{106}}{106}$

68. $\cos(\theta) = -\dfrac{2\sqrt{5}}{25}$, $\sin(\theta) = -\dfrac{11\sqrt{5}}{25}$

69. $r = 1.125$ inches, $\omega = 9000\pi \, \frac{\text{radians}}{\text{minute}}$, $x = 1.125\cos(9000\pi\, t)$, $y = 1.125\sin(9000\pi\, t)$. Here x and y are measured in inches and t is measured in minutes.

70. $r = 28$ inches, $\omega = \frac{2\pi}{3} \frac{\text{radians}}{\text{second}}$, $x = 28\cos\left(\frac{2\pi}{3}\,t\right)$, $y = 28\sin\left(\frac{2\pi}{3}\,t\right)$. Here x and y are measured in inches and t is measured in seconds.

71. $r = 1.25$ inches, $\omega = 14400\pi \frac{\text{radians}}{\text{minute}}$, $x = 1.25\cos(14400\pi\,t)$, $y = 1.25\sin(14400\pi\,t)$. Here x and y are measured in inches and t is measured in minutes.

72. $r = 64$ feet, $\omega = \frac{4\pi}{127} \frac{\text{radians}}{\text{second}}$, $x = 64\cos\left(\frac{4\pi}{127}\,t\right)$, $y = 64\sin\left(\frac{4\pi}{127}\,t\right)$. Here x and y are measured in feet and t is measured in seconds